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The Optimal Design of Fallible Organizations: Invariance of Optimal Decision Criterion and Uniqueness of Hierarchy and Polyarchy Structures

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Abstract We present a general framework to study the project selection problem in an organization of fallible decision-makers. We show that when the organizational size and the majority rule for project acceptance are optimized *simultaneously*, the optimal quality of decision-making, as determined by the decision criterion, is *invariant*, and depends only on the expertise of decision-makers. This result clarifies that the circumstances under which the decision-making quality varies with the organizational structure are situations where the organizational size or majority rule is restricted from reaching the optimal level. Moreover, in contrast to earlier findings in the literature that the hierarchy and the polyarchy are generally sub-optimal structures, we show that when the size, structure and decision criterion are simultaneously optimized, the hierarchy and the polyarchy are in fact *the only possible* optimal organizational structures when decision-making costs are present.

1 Introduction

This paper studies the optimal design of an organization in which a team of fallible decision-makers collectively decides whether to accept or reject investment projects. We consider the case where project evaluation takes place simultaneously in a committee of identically skilled decision-makers. Each decision-maker observes a signal about the quality of the investment project and endogenously selects a decision criterion (i.e. the project evaluation standard), such that if the signal exceeds

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the decision criterion, he or she votes to accept the project; otherwise, the decision-maker votes to reject the project. The expertise of a decision-maker is measured by the relative likelihoods of correctly selecting good projects and rejecting bad projects. Based on the voting profile, the organization's decision problem is to accept or reject each project based on a pre-determined majority rule. Our objective is to study the optimal relationship between the *structure* of the organization—as defined by its size and the majority rule for project acceptance—and the *quality* of decision-making—as determined by the choice of the decision criterion.

There are several contributions of this paper. Firstly, we generalize the project selection framework presented in Ben-Yashar and Nitzan (1998), using the concept of monotone likelihood ratio condition to model the notion that a decision-maker is more likely to observe a signal of higher value for a good project than for a bad project.

Secondly, and more importantly, we extend the analysis beyond Ben-Yashar and Nitzan (1998) to consider the simultaneous optimal choice of the organizational size, the majority rule for project acceptance and the decision criterion. This problem was not considered in Ben-Yashar and Nitzan (1998). We show that when the structure and quality of organizational decision-making are selected optimally, the optimal decision criterion is in fact *invariant* with respect to the size of the organization, the majority acceptance rule, as well as the quality of the investment environment (Proposition 1). The optimal quality of organizational decision-making is only a function of the expertise of the decision-makers (to be explained in Section 2) when the optimal organizational structure is adopted. The implication of this result is that the optimal design of the organization can be conducted as a two-step process: first, determine the optimal decision criterion, and then choose the optimal combination of organizational size and project acceptance majority rule.

Thirdly, the invariance property of the optimal decision criterion has two important implications on the optimal organizational structure. One implication is that organizations of different sizes perform equally well in terms of the expected *gross* project payoff, if the optimal decision criterion is adopted. However, when there are fixed costs in employing additional decision-makers, the optimal organizational structure will be the smallest feasible one that maximizes the organization's expected *net* payoff. Provided the decision criterion is set at the *invariant* optimal level, there are only two possible types of optimal organizational structures: the hierarchy (where full consensus is required for acceptance) for a mediocre investment environment, and the polyarchy (where the support of one decision-maker is sufficient) for an above-average investment environment (Proposition 2).

This result has interesting implications when related to the analysis in Ben-Yashar and Nitzan (2001) and Koh (2005), which studied the optimality of the hierarchical and polyarchical decision structures.[1] Both papers demonstrated the fragility of the hierarchy and polyarchy as optimal organizational structures when

[1] The literature on fallible collective decision-making includes the seminal work of Nitzan and Paroush (1985), Sah and Stiglitz (1986, 1988), Paroush and Karotkin (1989), as well as more recent work of Sah (1990, 1991), Koh (1992, 1994a,b, 2005), Ben-Yashar and Nitzan (1997, 2001) and Ben-Yashar and Paroush (2001). The strategic aspects of collective decision-making have been studied by Austen-Smith and Banks (1996), Feddersen and Pesedorfer (1998), Dekel and Piccione (2000), Li, Rosen and Suen (2001), and Persico (2004).

the decision criterion is *fixed* at a level that is not necessarily the optimal level. However, as we show in this paper, *if* the decision criterion is set at the optimal level and there are fixed costs to enlarging the organization, the hierarchy and polyarchy structures are in fact the unique optimal organizational structures.

The rest of the paper is organized as follows. In Section 2, we generalize the project selection model of Ben-Yashar and Nitzan (1998). In Section 3, we examine different aspects of the optimal organizational design—the decision criterion, the majority rule for project acceptance, and the organizational size. In Section 4, we present the results in Propositions 1 and 2 and discuss their relationships to Ben-Yashar and Nitzan (1998, 2001) and Koh (2005). Section 5 concludes the paper with a discussion of the results.

2 A generalization of the project selection model

We first present a generalization of the project selection framework described in Ben-Yashar and Nitzan (1998). This generalization is not a trivial exercise. We shall show that if the information structure satisfies the monotone likelihood ratio condition (to be described shortly), this is sufficient to generate the results in Ben-Yashar and Nitzan (1998), which relied on stronger assumptions of the information structure to obtain their results. The generalized framework presented here shows that the results of Ben-Yashar and Nitzan (1998) are valid in a more general setting.

Consider an organization of n members whose objective is to maximize the expected payoff from selecting and implementing projects. There are two types of projects: good (G) and bad (B). For each project, there are two possible decisions: accept (A) or reject (R). Let Q (=G or B) denote the state of a project, while D (=A or R) denote the decision on the project. The expected payoff associated with the decision on a particular project is $\Pi(D|Q)$. Clearly, we require that $\Pi(A|G)>\Pi(A|B)$ and $\Pi(R|B)\geq\Pi(R|G)$, so that there is an optimal action associated with each type of project. Let $\Pi(G)\equiv\Pi(A|G)-\Pi(R|G)$, and $\Pi(B)\equiv\Pi(R|B)-\Pi(A|B)$. The proportion of good projects in the project pool is assumed to be fixed at α, where $0<\alpha<1$.

The expertise of decision-makers is modeled as follows. We assume that decision-makers can differentiate good projects from bad projects, but only imperfectly in the following sense. When a decision-maker evaluates a project, he or she observes a signal r about each project, where $r\in[\underline{r},\bar{r}]$. Let $h(r|Q)$ and $H(r|Q)$ denote, respectively, the density function and conditional distribution function for a signal, conditional on the project being of quality Q. We assume that both $h(r|Q)$ and $H(r|Q)$ are continuously differentiable, and that $h(r|Q)$ satisfies the monotone likelihood ratio condition (MLRC). The MLRC property means that for two signals r_1 and r_2, where $r_1>r_2$,

$$\frac{h(r_1|G)}{h(r_2|G)}>\frac{h(r_1|B)}{h(r_2|B)} \tag{1}$$

Therefore, a decision-maker is more likely to observe a more favorable signal r_1 compared with r_2, when the project is a good project (i.e. of quality G), then if it is a bad project (i.e. of quality B). We require the following lemma for our analysis.

Lemma 1 If $h(r|Q)$ satisfies the monotone-likelihood ratio condition,

$$[a]\frac{h(r|B)}{1-H(r|B)} > \frac{h(r|G)}{1-H(r|G)}; [b]\frac{h(r|G)}{H(r|G)} > \frac{h(r|B)}{H(r|B)}; [c]H(r|G) < H(r|B).$$

The proof of Lemma 1 is given in Appendix A. Lemma 1[c] implies that $H(r|G)$ dominates $H(r|B)$ in terms of first-order stochastic dominance, and was also shown in Proposition 1 of Milgrom (1981).

All projects are indistinguishable to the decision-makers, ex ante, before undergoing any evaluation. Decision-makers evaluate the project independently, and based on their assessment (which is captured in the signal r), communicate a binary report ("Yes" or "No"), summarizing their opinion on the appropriate action to take on a project. Let s=1 denote a "Yes" vote, which is a recommendation to accept the project; similarly, s=0 denotes a "No" vote, which is a recommendation to reject the project. In deciding how to vote for a project, each decision-maker selects a decision criterion (i.e. cutoff point) θ so that if the signal r is greater (less) than θ, the decision-maker votes 'Yes' ('No'). A decision-maker's choice of θ therefore affects the overall quality of organizational decision-making.

The probability that a decision-maker will give a positive review on a project is $P(1|Q)=1-H(\theta|Q)$. The probability of a project receiving a bad review is $P(0|Q)=H(\theta|Q)$. Decision-making ability is imperfect in the sense that $P(1|G)<1$ and $P(1|B)>0$, but discriminatory in the sense that $P(1|G)>P(1|B)$.

For a given decision criterion, θ, a measure of the decision-maker's expertise to screen and select good projects is given by the following log-likelihood ratio,

$$\varepsilon_A \equiv \ln[1-H(\theta|G)] - \ln[1-H(\theta|B)] \tag{2}$$

Similarly, the expertise to discriminate and reject bad projects is measured by

$$\varepsilon_R \equiv \ln H(\theta|B) - \ln H(\theta|G) \tag{3}$$

Since $H(\theta|G)<H(\theta|B)$, it follows that $\varepsilon_A>0$ and $\varepsilon_R>0$. By Lemma 1, ε_A is increasing in θ, while ε_R is decreasing in θ. Raising the decision criterion θ improves the expertise for selecting good projects but the expertise to screen out bad projects suffers.

3 Designing the optimal organization

In this section, we derive the results necessary for the construction of the optimal organization. For a given organizational size of n, a project is accepted only if it receives at least k positive reviews. Let $\tilde{\theta} = (\theta_1, \ldots, \theta_n)$ denote the set of decision criteria. The probability that a project will be accepted is

$$P(\tilde{\theta}, k, n, Q) = \sum_{j=k}^{n} C_j^n \prod_{i \in S_j} [1 - H(\theta_i|Q)] \prod_{i \notin S_j} H(\theta_i|Q) \tag{4}$$

where S_j is a subset of j (out of n) decision-makers that vote "Yes" for the project. For each project evaluated, the unconditional expected utility payoff to the organization is

$$V\left(\tilde{\theta}, k, n\right) = \alpha P\left(\tilde{\theta}, k, n, G\right)\Pi(G) - (1-\alpha)P\left(\tilde{\theta}, k, n, B\right)\Pi(B) \tag{5}$$

There are several dimensions in designing the optimal organization for project evaluation: the decision criteria, the optimal majority rule for project acceptance and the optimal organizational size. We consider each aspect in turn.

3.1 Selecting the decision criterion

For a given organizational structure, k and n, each decision-maker i independently chooses a decision criterion (i.e. cutoff point), taking the decision criteria of other decision-makers as given. Making use of the following relationship,

$$\begin{aligned} P\left(\tilde{\theta}, k, n, Q\right) &= H(\theta_i|Q)P\left(\tilde{\theta}_{-i}, k, n-1, Q\right) \\ &\quad + [1 - H(\theta_i|Q)]P\left(\tilde{\theta}_{-i}, k-1, n-1, Q\right) \end{aligned}$$

where $\tilde{\theta}_{-i} = (\theta_1, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_n)$, the partial derivative of $V\left(\tilde{\theta}, k, n\right)$ with respect to θ_i is given by

$$\begin{aligned} \frac{\partial V\left(\tilde{\theta}, k, n\right)}{\partial \theta_i} &= \alpha\Pi(G)h(\theta_i|G)\left[P\left(\tilde{\theta}_{-i}, k-1, n-1, G\right) - P\left(\tilde{\theta}_{-i}, k, n-1, G\right)\right] \\ &\quad -(1-\alpha)\Pi(B)h(\theta_i|B)\left[P\left(\tilde{\theta}_{-i}, k-1, n-1, B\right)\right. \\ &\quad \left. -P\left(\tilde{\theta}_{-j}, k, n-1, B\right)\right] \end{aligned}$$

Let $\tilde{\theta}^* = \left(\theta_1^*, ..., \theta_n^*\right)$ denote the set of optimal decision criteria, and $\tilde{\theta}^*_{-i} = \left(\theta_1^*, ..., \theta_{i-1}^*, \theta_{i+1}^*, ..., \theta_n^*\right)$. The first-order condition for decision-maker i's optimal decision criterion, denoted θ_i^*, and evaluated at $\tilde{\theta}^*$, is given by

$$\begin{aligned} \left.\frac{\partial V\left(\tilde{\theta}, k, n\right)}{\partial \theta_i}\right|_{\tilde{\theta}^*} &= \alpha\Pi(G)h\left(\theta_i^*|G\right)\left[P\left(\tilde{\theta}^*_{-i}, k-1, n-1, G\right)\right. \\ &\quad \left. -P\left(\tilde{\theta}^*_{-i}, k, n-1, G\right)\right] - (1-\alpha)\Pi(B)h\left(\theta_i^*|B\right) \\ &\quad \times\left[P\left(\tilde{\theta}^*_{-i}, k-1, n-1, B\right) - P\left(\tilde{\theta}^*_{-i}, k, n-1, B\right)\right] = 0 \end{aligned} \tag{6}$$

This leads to the following result:

$$\beta = \frac{h(\theta_i^*|B)\left[P(\tilde{\theta}^*_{-i}, k-1, n-1, B) - P(\tilde{\theta}^*_{-i}, k, n-1, B)\right]}{h(\theta_i^*|G)\left[P(\tilde{\theta}^*_{-i}, k-1, n-1, G) - P(\tilde{\theta}^*_{-i}, k, n-1, G)\right]} \tag{7}$$

where

$$\beta \equiv \frac{\alpha \Pi(G)}{(1-\alpha)\Pi(B)} \tag{8}$$

is a measure of the quality of the investment environment.[2] Similarly, we can show that the necessary second-order condition for θ_i^*, evaluated at $\tilde{\theta}^*$, is given by

$$\begin{aligned} \left.\frac{\partial V^2(\tilde{\theta}, k, n)}{\partial \theta_i^2}\right|_{\tilde{\theta}^*} &= \alpha \Pi(G) h'(\theta_i^* | G)\left[P(\tilde{\theta}^*_{-i}, k-1, n-1, G) - P(\tilde{\theta}^*_{-i}, k, n-1, G)\right] \\ &= -(1-\alpha)\Pi(B) h'(\theta_i^* | B)\left[P(\tilde{\theta}^*_{-i}, k-1, n-1, B)\right. \\ &\quad \left. - P(\tilde{\theta}^*_{-i}, k, n-1, B)\right] < 0 \end{aligned} \tag{9}$$

where $h'(r|Q) \equiv \frac{\partial h(r|Q)}{\partial r}$. Since decision-makers are assumed to be identically skilled, a symmetric equilibrium θ^* exists, and accordingly, will be the focus of our analysis henceforth. Hence, utilizing the relationship that

$$\begin{aligned} &P(\tilde{\theta}^*_{-i}, k, n-1, Q) - P(\tilde{\theta}^*_{-i}, k-1, n-1, Q) \\ &\quad = -C_{k-1}^{n-1}[1 - H(\theta^*|Q)]^{k-1} H(\theta^*|Q)^{n-k} \end{aligned}$$

this implies that at the symmetric equilibrium θ^*, the condition in Eq. 7 can be rewritten as

$$\beta = \frac{h(\theta^*|B)[1 - H(\theta^*|B)]^{k-1} H(\theta^*|B)^{n-k}}{h(\theta^*|G)[1 - H(\theta^*|G)]^{k-1} H(\theta^*|G)^{n-k}} \tag{10}$$

Similarly, the necessary second-order condition for the optimal θ^*, in Eq. 9, can be rewritten as

$$\beta h'(\theta^*|G) > h'(\theta^*|B) \frac{[1 - H(\theta^*|B)]^{k-1} H(\theta^*|B)^{n-k}}{[1 - H(\theta^*|G)]^{k-1} H(\theta^*|G)^{n-k}} \tag{11a}$$

Substituting the relationship in Eq. 10 into Eq. 11a leads to the following condition:

$$\frac{h'(\theta^*|G)}{h(\theta^*|G)} > \frac{h'(\theta^*|B)}{h(\theta^*|B)} \tag{11b}$$

Using Lemma 1 and the condition for θ^* in Eq. 11a, we take natural logs on both sides of Eq. 10, and differentiate with respect to k and n in turn to obtain the following results:

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}k} = Z\left[\ln \frac{H(\theta^*|B)}{1 - H(\theta^*|B)} - \ln \frac{H(\theta^*|G)}{1 - H(\theta^*|G)}\right] > 0 \tag{12a}$$

[2] The investment environment is said to be above average (mediocre) if β is greater (less) than 1. When β=1, the investment environment is said to be of neutral quality.

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}n} = Z[\ln H(\theta^*|G) - \ln H(\theta^*|B)] < 0 \tag{12b}$$

where

$$Z^{-1} \equiv \frac{h'(\theta^*|B)}{h(\theta^*|B)} - \frac{h'(\theta^*|G)}{h(\theta^*|G)} + (k-1)\left[\frac{h(\theta^*|G)}{1-H(\theta^*|G)} - \frac{h(\theta^*|B)}{1-H(\theta^*|B)}\right]$$
$$+(n-k)\left[\frac{h(\theta^*|B)}{H(\theta^*|B)} - \frac{h(\theta^*|G)}{H(\theta^*|G)}\right] < 0$$

Let us now relate the results in Eqs. 12a and 12b to Theorems 1 and 2 in Ben-Yashar and Nitzan (1998). Using the notation in the present paper, the following sufficiency conditions were assumed regarding the information structure in Ben-Yashar and Nitzan (1998):

$$\frac{\mathrm{d}[1-H(\theta|G)]}{\mathrm{d}\theta} < 0; \frac{\mathrm{d}^2[1-H(\theta|G)]}{\mathrm{d}\theta^2} < 0; \frac{\partial^2 P(\tilde{\theta},k,n,G)}{\partial[1-H(\theta|G)]^2} < 0$$

$$\frac{\mathrm{d}H(\theta|B)}{\mathrm{d}\theta} > 0; \frac{\mathrm{d}^2 H(\theta|B)}{\mathrm{d}\theta^2} < 0; \frac{\partial^2[1-P(\tilde{\theta},k,n,B)]}{\partial H(\theta|B)^2} < 0$$

Under these sufficiency conditions, Theorem 1 of Ben-Yashar and Nitzan (1998) states that $\frac{\mathrm{d}\theta^*}{\mathrm{d}\kappa} > 0$ where $\kappa \equiv kn$, while Theorem 2 of Ben-Yashar and Nitzan (1998) states that $\frac{\mathrm{d}\theta^*}{\mathrm{d}n} > 0$, when certain conditions hold. Since $\frac{\mathrm{d}\kappa}{\mathrm{d}n} = \frac{1}{n} > 0$, Theorem 1 of Ben-Yashar and Nitzan (1998) is equivalent to our result in Eq. 12a, which is obtained only under the assumption that the informational structure satisfy the MLRC property. Furthermore, the MLRC property is also a necessary and sufficient condition to obtain the result in Eq. 12b. Hence, Theorem 2 of Ben-Yashar and Nitzan (1998) is also valid in a more general setting.

3.2 Optimal majority rule for project acceptance

Next, we consider the selection of the optimal majority rule, k, for accepting a project. Since our focus is on the symmetric optimal θ, we replace $\tilde{\theta}$ with θ in the functions $V(\theta,k,n)$ and $P(\theta,k,n,Q)$, henceforth.

Lemma 2 For a given n, $V(\theta,k,n)$ achieves a global maximum at either one value of k or two adjacent values of k.

(The proof of Lemma 2 is given in Appendix B.) For a given θ and n, let k^* denote the optimal majority rule for project acceptance, where k^* satisfies

$V(\theta,k^*,n)\geq V(\theta,k^*-1,n)$ and $V(\theta,k^*,n)\geq V(\theta,k^*+1,n)$. This translates into the following optimality condition for k^*:

$$\frac{[1-H(\theta|B)]^{k^*}H(\theta|B)^{n-k^*}}{[1-H(\theta|G)]^{k^*}H(\theta|G)^{n-k^*}}\leq\beta\leq\frac{[1-H(\theta|B)]^{k^*-1}H(\theta|B)^{n-k^*+1}}{[1-H(\theta|G)]^{k^*-1}H(\theta|G)^{n-k^*+1}} \tag{13}$$

The solution of k^* has an explicit form. Taking natural logs on both sides of Eq. 13 and rearranging, we obtain

$$\Gamma(\theta,n,\beta)\leq k^*\leq\Gamma(\theta,n,\beta)+1 \tag{14}$$

where $\Gamma(\theta,n,\beta)\equiv\frac{n\varepsilon_R-\ln\beta}{\varepsilon_A+\varepsilon_R}$. If $\Gamma(\theta,n,\beta)$ is an integer, $k^*=\Gamma(\theta,n,\beta)$ or $\Gamma(\theta,n,\beta)+1$, or both. If $\Gamma(\theta,n,\beta)$ is not an integer, then k^* is the integer that lies between $\Gamma(\theta,n,\beta)$ and $\Gamma(\theta,n,\beta)+1$. Letting $k^*\cong\Gamma(\theta,n,\beta)$, we obtain

$$\frac{\partial k^*}{\partial n}=\frac{\varepsilon_R}{\varepsilon_A+\varepsilon_R}\in(0,1). \tag{15}$$

3.3 Optimal organizational size

Since k^* is increasing in n, it is easy to see that for a given k, $V(\theta,k,n)$ is single-peaked in n, and there are at most two values of n, adjacent to each other, that maximizes $V(\theta,k,n)$. A formal proof of this single-peak property is given in Appendix C. Let n^* denote the optimal organizational size for a given θ and k, where $V(\theta,k,n^*)>V(\theta,k,n^*-1)$ and $V(\theta,k,n^*)>V(\theta,k,n^*+1)$. This yields the optimality condition:

$$\frac{[1-H(\theta|B)]^{k}H(\theta|B)^{n^*-k}}{[1-H(\theta|G)]^{k}H(\theta|G)^{n^*-k}}\leq\beta\leq\frac{[1-H(\theta|B)]^{k}H(\theta|B)^{n^*-k+1}}{[1-H(\theta|G)]^{k}H(\theta|G)^{n^*-k+1}} \tag{16}$$

which leads to the solution for n^*, characterized as follows:

$$\Delta(\theta,k,\beta)-1\leq n^*\leq\Delta(\theta,k,\beta) \tag{17}$$

where $\Delta(\theta,k,\beta)\equiv\frac{(\varepsilon_A+\varepsilon_R)k+\ln\beta}{\varepsilon_R}$. If $\Delta(\theta,k,\beta)$ is an integer, then n^* is $\Delta(\theta,k,\beta)$ or $\Delta(\theta,k,\beta)+1$, or both. If $\Delta(\theta,k,\beta)$ is not an integer, then n^* is the integer that lies between $\Delta(\theta,k,\beta)$ and $\Delta(\theta,k,\beta)+1$.

4 Simultaneous optimal choice of θ, k and n

With the results obtained in Section 3, we are ready to derive the optimal organizational structure, where θ, k and n are optimized simultaneously. In this case, all the optimality conditions in Eqs. 10, 11a, 13, 14, 16 and 17 must hold simultaneously, to characterize the optimal organizational structure $\{\theta^*, k^*, n^*\}$. By substituting the optimality condition for θ^* in Eq. 10 into Eq. 13, and simplifying, we obtain

$$\frac{1-H(\theta^*|B)}{1-H(\theta^*|G)} \leq \frac{h(\theta^*|B)}{h(\theta^*|G)} \leq \frac{H(\theta^*|B)}{H(\theta^*|G)} \tag{18}$$

Similarly, substituting Eq. 10 into Eq. 16 and simplifying, we obtain

$$\frac{1-H(\theta^*|B)}{1-H(\theta^*|G)}\frac{h(\theta^*|B)}{h(\theta^*|G)} \leq \frac{[1-H(\theta^*|B)]H(\theta^*|B)}{[1-H(\theta^*|G)]H(\theta^*|G)} \tag{19}$$

Since $H(\theta^*|B)>H(\theta^*|G)$, the optimality condition in Eq. 19 is more binding than in Eq. 18. Together with Eq. 11b, this leads to the following condition for the choice of the optimal decision criterion θ^*:

$$\text{Max}\left\{\frac{h'(\theta^*|B)}{h'(\theta^*|G)}, \frac{1-H(\theta^*|B)}{1-H(\theta^*|G)}\right\} \leq \frac{h(\theta^*|B)}{h(\theta^*|G)} \leq \left[\frac{1-H(\theta^*|B)}{1-H(\theta^*|G)}\right]\frac{H(\theta^*|B)}{H(\theta^*|G)} \tag{20}$$

Next, the conditions in Eqs. 14 and 17 can be combined to yield the optimality condition for the simultaneous choice of k^* and n^*:

$$\frac{n^*\hat{\varepsilon}_R - \ln\beta}{\hat{\varepsilon}_A + \hat{\varepsilon}_R} \leq k^* \leq \frac{(n^*+1)\hat{\varepsilon}_R - \ln\beta}{\hat{\varepsilon}_A + \hat{\varepsilon}_R} \tag{21}$$

where $\hat{\varepsilon}_A \equiv \ln[1-H(\theta^*|G)] - \ln[1-H(\theta^*|B)]$. This leads to our first main result.

Proposition 1 The optimal design of the organization can be conducted as a two-step process. First, select the optimal decision criterion θ^* to satisfy the constraint in Eq. 20, and then proceed to select k^* and n^* jointly to satisfy Eq. 21.

Proposition 1 indicates that while the optimal size, n^*, and the optimal majority rule, k^*, varies with the investment environment β, in a specific manner, as given in Eq. 21, the optimal decision criterion, θ^*, is invariant with respect to β. This result can be easily derived from the total differentiation of θ^* with respect to β:

$$\frac{d\theta^*}{d\beta} = \left\{\frac{\partial\theta^*}{\partial k}\frac{\partial k^*}{\partial n} + \frac{\partial\theta^*}{\partial n}\right\}\frac{\partial n^*}{\partial\beta} = 0 \tag{22}$$

where

$$\frac{\partial\theta^*}{\partial k} = Z(\hat{\varepsilon}_A + \hat{\varepsilon}_R), \frac{\partial\theta^*}{\partial n} = -Z\hat{\varepsilon}_R, \frac{\partial k^*}{\partial n} = \frac{\hat{\varepsilon}_R}{\hat{\varepsilon}_A + \hat{\varepsilon}_R} \tag{23}$$

with Z defined earlier in the derivation of Eqs. 12a and 12b. This result implies that while the optimal decision criterion θ^* decreases with k and increases with n—as we showed earlier in Eqs. 12a and 12b—the net effect on θ^*, when k^* and n^* are chosen optimally, is zero.

Moreover, from Eq. 21, it follows from the invariance property of the optimal decision criterion θ^* that the optimal organizational structure (k^*, n^*) is not unique.

Therefore, when the decision-making quality is optimally set at θ*, small organizations can perform just as well as large organizations when the optimal majority rule corresponding to the organizational size is adopted. Therefore, if there are *no costs* incurred in enlarging the decision-making team, the optimal organizational size is not unique.

However, if there are costs incurred in enlarging the decision-making team, there will be a unique optimal organizational size. In this case, the unique optimal organizational size will be the smallest feasible organization that maximizes the expected net project payoff. In fact, we can show that when β<1, the unique optimal organization is a hierarchy where k*=n*. Similarly, when β>1, the polyarchy, where k*=1, is the unique optimal organization.

First, consider the hierarchy, where k=n. Using the result in Eq. 17, the hierarchy is an optimal organizational structure if $\frac{-\ln\beta}{\varepsilon_A} \leq n^H \leq \frac{\varepsilon_R - \ln\beta}{\varepsilon_A}$, where n^H denotes the optimal size of the hierarchy, where ε_A and ε_R are defined in Eqs. 2 and 3. In order that n^H>0, we must have β<1. Similarly, in the case of the polyarchy, where k=1, this is an optimal organizational structure if $\frac{\varepsilon_A + \ln\beta}{\varepsilon_R} \leq n^P \leq \frac{\varepsilon_A + \ln\beta}{\varepsilon_R} + 1$, where n^P denotes the optimal size of the polyarchy. The necessary condition for n^P to be positive is that β>1. It is easy to see that if the organizational size is restricted below n^H, the hierarchy is dominated by a decision rule to always reject projects. Similarly, if the organizational size is restricted below n^P, then the polyarchy is dominated by a decision rule to always accept projects. Hence, n^H and n^P denote the smallest feasible optimal organizational size for β<1 and β>1, respectively.[3] We thus obtain.

Proposition 2 When fixed costs are present in enlarging the organization, and the decision criterion is chosen optimally, the unique optimal organizational structure is a hierarchy when β<1, and a polyarchy when β>1.

Proposition 2 has interesting implications when related to the findings in Ben-Yashar and Nitzan (2001) and Koh (2005). Both papers showed that the hierarchy and polyarchy can exist as optimal organizational structures only under specific conditions regarding the investment environment. By contrast, we have found in our study that the hierarchy and the polyarchy are in fact the unique optimal organizational structures, when there are costs involved in enlarging the organization.

The two sets of results complement each other, and apply in different settings. In the setting considered in Ben-Yashar and Nitzan (2001) and Koh (2005), which focuses on sequential decision processes, the decision criterion is fixed at a level that is not necessarily the optimal level. As a result, changes in the investment environment will necessitate optimal adjustments in both the organizational size and the majority rule for acceptance to compensate for the fixed decision-making quality. By contrast, in the present study, we allow all the elements of an organization—size, majority rule and the decision criterion—to be simultaneously optimized. With the decision criterion set at the optimal level (defined by the abilities of the decision-makers), the hierarchy and the polyarchy turn out to be the only optimal organizational structures that can exist when there are costs to enlarging the organization.

[3] It follows from the result in Eq. 14 that if the size of the organization, n, is larger than n^H when β<1, k* will be less than n. Similarly, if n>n^P when β>1, k* will be greater than one.

The implication of these results is that the circumstances under which the *quality* of organizational decision-making will vary with the organizational size and the project acceptance rule are situations where either or both of these aspects of the organizational *structure* are restricted from reaching their optimal levels, and the decision criterion is fixed at a sub-optimal level.

5 Conclusion

In this paper, we present a generalization of the project selection framework in Ben-Yashar and Nitzan (1998), and extend the analysis to consider the simultaneous optimal choice of the organizational size, quality majority rule and the decision criterion. We demonstrate that when the structure and quality of organizational decision-making are simultaneously optimized, the optimal decision criterion is *invariant* with respect to the organizational size and majority rule, and is only a function of the abilities of the decision-makers. As a result, the optimal design of the organization can be conducted as a two-step process: first, determine the optimal decision criterion, and then choose the optimal organizational size and the project acceptance majority rule.

The invariance property of the optimal decision criterion implies that in the absence of fixed costs, organizations of different sizes can be structured to yield the same gross expected payoffs, if the optimal decision criterion and the optimal majority rule for project acceptance are adopted. More importantly, if there are fixed costs in employing additional decision-makers, and provided the decision criterion is set at the *invariant* optimal level, there are only two possible types of optimal organizational structures to adopt: the hierarchy (where full consensus is required for acceptance) for a mediocre investment environment, and the polyarchy (where the support of one decision-maker is sufficient) for an above-average investment environment.

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Appendix A

Proof of Lemma 1

[a] By the definition of MLRC, we have $\frac{h(x|G)}{h(r|G)} > \frac{h(x|B)}{h(r|B)}$ if $x > r$. Integrating over the interval $x \in [r, \bar{r}]$, we have $\frac{1}{h(r|G)} \int_r^{\bar{r}} h(x|G)dx > \frac{1}{h(r|B)} \int_r^{\bar{r}} h(x|B)dx$. This leads to $\frac{1-H(r|G)}{h(r|G)} > \frac{1-H(r|B)}{h(r|B)}$.

[b] Similarly, by MLRC, we have $\frac{h(r|G)}{h(z|G)} > \frac{h(r|B)}{h(z|G)}$ for $z<r$. Re-arranging, we obtain $\frac{h(z|B)}{h(r|B)} > \frac{h(z|G)}{h(r|G)}$ for $z<r$. Hence, $\frac{1}{h(r|B)} \int_{\underline{r}}^{r} h(z|B)dz > \frac{1}{h(r|G)} \int_{\underline{r}}^{r} h(z|G)dz$ which leads to $\frac{H(r|B)}{h(r|B)} > \frac{H(r|G)}{h(r|G)}$.

[c] From Lemmas 1[a] and 1[b] , we have $\frac{H(r|B)}{H(r|G)} > \frac{h(r|B)}{h(r|G)} > \frac{1-H(r|B)}{1-H(r|G)}$. This in turn implies that $H(r|G)<H(r|B)$, which is the result obtained in Milgrom (1981, pp 383).

Q.E.D.

Appendix B

Proof of Lemma 2 Using the following relationship when the decision criterion is the same for all decision-makers, we obtain

$$P(\theta, k, n, Q) - P(\theta, k-1, n, Q) = -C_{k-1}^{n}[1 - H(\theta|Q)]^{k-1} H(\theta|Q)^{n-k+1}$$

Therefore,

$$\begin{aligned} V(\theta, k, n) - V(\theta, k-1, n) = & -\alpha \Pi(G) C_{k-1}^{n}[1 - H(\theta|G)]^{k-1} H(\theta|G)^{n-k+1} \\ & +(1-\alpha)\Pi(B) C_{k-1}^{n}[1 - H(\theta|B)]^{k-1} H(\theta|B)^{n-k+1} \end{aligned}$$

Consider the following function:

$$\Phi(k, n) \equiv V(\theta, k, n) - V(\theta, k-1, n) - \Omega(k, n)[V(\theta, k+1, n) - V(\theta, k, n)]$$

where

$$\Omega(k, n) \equiv \frac{k}{n-k+1} \times \left[\frac{\alpha[1 - H(\theta|G)]^{k-1} H(\theta|G)^{n-k+1} + (1-\alpha)[1 - H(\theta|B)]^{k-1} H(\theta|B)^{n-k+1}}{\alpha[1 - H(\theta|G)]^{k} H(\theta|G)^{n-k} + (1-\alpha)[1 - H(\theta|B)]^{k} H(\theta|B)^{n-k}}\right]$$

Straightforward manipulation allows us to rewrite $\Phi(k, n)$ as follows:

$$\Phi(k, n) \equiv \Psi(k, n)\left[\frac{H(\theta|B)}{1 - H(\theta|B)} - \frac{H(\theta|G)}{1 - H(\theta|G)}\right]$$

where

$$\Psi(k) \equiv \frac{\alpha(1-\alpha) C_{k-1}^{n}[\Pi(G) + \Pi(B)][1 - H(\theta|B)]^{k} H(\theta|B)^{n-k}[1 - H(\theta|G)]^{k} H(\theta|G)^{n-k}}{\alpha[1 - H(\theta|G)]^{k} H(\theta|G)^{n-k} + (1-\alpha)[1 - H(\theta|B)]^{k} H(\theta|B)^{n-k}}$$

By Lemma 1, we have $\frac{H(\theta|G)}{1-H(\theta|G)} < \frac{H(\theta|B)}{1-H(\theta|B)}$, so that it must be the case that $\Phi(k, n) > 0$. Therefore, the following relationships must hold:

$$\begin{aligned} V(\theta, k, n) \geq V(\theta, k-1, n) &\Rightarrow V(\theta, k-1, n) > V(\theta, k-2, n) \\ V(\theta, k, n) \geq V(\theta, k+1, n) &\Rightarrow V(\theta, k+1, n) > V(\theta, k+2, n) \end{aligned}$$

This completes the proof that $V(\theta, k, n)$ is single-peaked in k.

Q.E.D.

Appendix C

Lemma 3 For a given k, $V(\theta,k,n)$ achieves a global maximum at either one value of n or two adjacent values of n.

Proof of Lemma 3 Consider the following difference:

$$\begin{aligned} V(\theta, k, n) - V(\theta, k, n-1) = {} & \alpha\Pi(G)C_{k-1}^{n-1}[1 - H(\theta|G)]^k H(\theta|G)^{n-k} \\ & -(1-\alpha)\Pi(B)C_{k-1}^{n-1}[1 - H(\theta|B)]^k H(\theta|B)^{n-k} \end{aligned}$$

Next, we construct the following function:

$$\Upsilon(k, n) \equiv [V(\theta, k, n+1) - V(\theta, k, n)] - \Theta(k, n)[V(\theta, k, n) - V(\theta, k, n-1)]$$

where

$$\Theta(k, n) \equiv \frac{k}{n-k+1} \times \left[\frac{\alpha[1 - H(\theta|G)]^k H(\theta|G)^{n-k+1} + (1-\alpha)[1 - H(\theta|B)]^k H(\theta|B)^{n-k+1}}{\alpha[1 - H(\theta|G)]^k H(\theta|G)^{n-k} + (1-\alpha)[1 - H(\theta|B)]^k H(\theta|B)^{n-k}}\right]$$

Straightforward computation yields

$$\Upsilon(k, n) \equiv \Psi(k, n)[H(\theta|G) - H(\theta|B)] < 0$$

where $\Psi(k, n)$ is defined in the proof of Lemma 2 in Appendix B. Hence, the following relationships hold:

$$\begin{aligned} V(\theta, k, n) \geq V(\theta, k, n-1) &\Rightarrow V(\theta, k, n-1) > V(\theta, k, n-2) \\ V(\theta, k, n) \geq V(\theta, k, n+1) &\Rightarrow V(\theta, k, n+1) > V(\theta, k, n+2) \end{aligned}$$

which implies that $V(\theta, k, n)$ is single-peaked in n.

Q.E.D.

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